ON THE RELATIONSHIP BETWEEN MEAN AND VARIANCE OF SOIL MOISTURE FIELDS

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ABSTRACT: The objective of this work was to explain an apparent contradiction in the literature related to the relationship between mean and variance (or standard deviation) of soil moisture fields. Some studies found an increase in soil moisture variance with decreasing mean soil moisture, while others showed a decrease. The evidence of maximum variance in the mid-range of mean soil moisture was also reported in the literature. In this paper, we focus on the effects of spatial variability of soil texture on the relationship between variance and mean of soil moisture during soil dry-down processes. Soil texture influences soil moisture mean and variance through its direct effects on evaporation and drainage, which are two main factors controlling soil drying. A differential equation describing soil moisture dry down is proposed and studied. Our study shows that as mean soil moisture is greater than a threshold, variance increases with decreasing mean soil moisture. If mean soil moisture is less than the threshold, variance decreases with decreasing mean soil moisture. The threshold depends on soil texture and is between the field capacity and the wilting point. The soil moisture dry-down equation is also applied to explain the apparent contradiction with regard to the relationship between mean and variance of soil moisture fields reported in the literature.

(KEY TERMS: spatial variability; soil moisture; soil texture; soil particle size distribution; evaporation; drainage; field capacity; wilting point.)


INTRODUCTION

Soil moisture is an important variable that affects atmospheric dynamics and hydrologic processes by influencing the partitioning of incoming radiation into sensible and latent heat fluxes and by separating precipitation into infiltration and surface runoff. Although soil moisture is an important variable, it is time consuming and labor intensive to measure soil moisture with a high sampling frequency and a large spatial coverage. Therefore, understanding of soil moisture spatial distributions may help us save time and effort to measure soil moisture.

Although spatial distributions of soil moisture have been widely studied, there is one contradiction in the literature related to the relationship between mean and variance of soil moisture fields, as summarized by Famiglietti et al. (1998). Several studies have shown that the variance of soil moisture decreases...
with decreasing mean soil moisture (Hills and Reynolds, 1969; Reynolds, 1970a,b,c; Henninger et al., 1976; Bell et al., 1980; Hawley et al., 1982; Robinson and Dean, 1993). Hupet and Vanclloster (2002) found that the variance of soil moisture increases with decreasing mean soil moisture. No systematic relationship between the variance and the mean soil moisture was found in other studies (Hawley et al., 1983; Charpentier and Groffman, 1992). Owe et al. (1982) found that soil moisture variance was maximum in the mid-range of mean soil moisture content. A normal distribution of surface soil moisture was found by Hills and Reynolds (1969); Bell et al. (1980); Hawley et al. (1983); Francis et al. (1986); Nyberg (1996). Famiglietti et al. (1998) indicated that the differences in climate, soils, vegetation, topography, and sampling frequency may contribute to the differences in those findings.

In this paper, we focus on the effects of the spatial variability of soil texture on the relationship between mean and variance of soil moisture. If we only study soil moisture measured at small scales where spatial variations of climate and vegetation can be ignored, the dominant factors controlling soil moisture distributions come from soils and topography. If soil moisture data are collected over relatively flat areas, the effects of topography can also be neglected. Soil texture controls soil moisture mean and variance through its direct effects on evaporation and drainage, which are two main factors controlling soil drying. In this study, we first developed a nonlinear differential equation to describe soil moisture dry down. We then applied this equation to study mean and variance of soil moisture and to explain the contradiction in the literature with regard to the relationship between mean and variance of soil moisture.

**SOIL MOISTURE DRY-DOWN EQUATION**

Evaporation and drainage are two dominant factors controlling soil moisture dry down. Many studies have shown that there are three stages of soil evaporation during soil dry-down process (e.g., Idso et al., 1974; Hillel, 1998). These three stages are as follows: stage I, which is atmosphere-controlled; stage II, which is soil hydraulic properties-controlled; and stage III, which is atmospheric demand controlled. To explain the dependency of the temporal behavior of the variance on mean soil moisture state, let us consider a dry-down process from saturated soil moisture ($\theta_s$) to residual soil moisture ($\theta_r$). Between those upper and lower limits of soil moisture states, we used two transition soil moisture states denoted by $\theta_1$ and $\theta_2$ to define the three stages of soil moisture dry-down process, which are stage I from $\theta_s$ to $\theta_1$, stage II from $\theta_1$ to $\theta_2$, and stage III from $\theta_2$ to $\theta_r$. These three stages can be illustrated in a plot of actual evaporation rate ($E$) vs. time (Hillel, 1998) (Figure 1a). According to Figure 1a, we can assume that in stage I the actual evaporation rate is equal to the potential rate (PE) and independent of the soil moisture. In the stage III, the actual evaporation rate is much less than the potential rate and approximately a constant. In the stage II, the actual evaporation rate is proportional to the soil moisture (Figure 1b). Therefore, we express the actual evaporation rate as follows:

$$E = \begin{cases} PE, & \theta_1 \leq \theta \leq \theta_s \text{ (stage I)} \\ c_1 \times \theta \times PE, & \theta_2 \leq \theta \leq \theta_1 \text{ (stage II)} \\ c_2 \times PE, & \theta_s \leq \theta \leq \theta_2 \text{ (stage III)} \end{cases}$$ (1)

For the continuity of the $E$ function at each transition point, we have

![Figure 1](image-url)
ON THE RELATIONSHIP BETWEEN MEAN AND VARIANCE OF SOIL MOISTURE FIELDS

\[ PE = c_1 \times \theta_1 \times PE \]
\[ c_1 \times \theta_2 \times PE = c_2 \times PE \]  

or

\[ c_1 = \frac{1}{\theta_1} \]
\[ c_2 = \frac{\theta_2}{\theta_1} \]  

Therefore,

\[ E = \begin{cases} 
PE, & \theta_1 \leq \theta \leq \theta_s \text{ (stage I)} \\
PE \times (\theta/\theta_1), & \theta_2 \leq \theta \leq \theta_1 \text{ (stage II)} \\
PE \times (\theta_2/\theta_1), & \theta_s \leq \theta \leq \theta_2 \text{ (stage III)} 
\end{cases} \]  

Although the concept of three stages of evaporation is easy to understand, how to define the transition points from stage I to stage II and from stage II to stage III is not easy. Idso et al. (1974) detected these transitions points through measuring surface albedo. As indicated by Hillel (1998), the transition from stage I to stage II is relatively easily detected because this transition is associated with a sharp decrease of evaporation. It is difficult to identify the transition from stage II to stage III because this transition is so gradual that stage II and stage III cannot be separated easily.

If we consider the physical process of evaporation, we understand that the rate of evaporation is the rate at which water molecules move from soil or water surface layer into the air above. There are several factors controlling evaporation, i.e., solar radiation, vapor pressure deficit, wind speed, soil hydraulic conductivity, and matric potential. It seems appropriate to use the matric potential rather than soil moisture to define the transition points among the three stages of evaporation, because the matric potential directly controls the evaporation rate, i.e., for large (small) negative pressure head, there are less (more) water molecules that can come from soil into the air. If we use \( \psi_1 \) or \( \psi_2 \) to denote the transition points, according to Clapp and Hornberger (1978), the corresponding soil moistures at \( \psi_1 \) or \( \psi_2 \) are

\[ \theta_{1,2} = \theta_s \left( \frac{|\psi_s|}{|\psi_{1,2}|} \right)^{1/b} \]  

where \( \psi_s \) and \( b \) are soil empirical parameters dependent on soil texture or soil particle size distribution (PSD). Different soil texture may have different transition points in term of soil moisture, but the same transition points in term of the pressure head. Using the pressure head to define the transition points is nothing new. In any hydrology textbook, we can find that pressure head at \(-330 \text{ cm}\) is used to define a state at which the remaining water held by surface tension on the soil particles is in equilibrium with the gravitational forces causing drainage, and \(-15,000 \text{ cm}\) is used to define a state at which the transpiration ceases and plants wilt (Dingman, 1994). These two states are called the field capacity, and the wilting point, respectively. On the other hand, in the literature (e.g., Davies and Allen, 1973; Federer, 1979, 1982; Spittlehouse and Black, 1981), the pressure heads at the field capacity and the wilting point are often used to represent our defined transition points \( \psi_1 \) and \( \psi_2 \). As the main objective of this paper was to study the relationship between mean and variance of soil moisture, we set \( \psi_1 \) or \( \psi_2 \) to be \(-330 \text{ cm}\) and \(-15,000 \text{ cm}\), respectively. The effects of the uncertainty in \( \psi_1 \) and \( \psi_2 \) on the relationship between mean and variance of soil moisture are beyond the scope of this paper.

There are many methods in the literature for estimating the potential evaporation rate, e.g., Penman-Monteith method (e.g., Jensen et al., 1990), Priestley-Taylor method (Priestley and Taylor, 1972), and temperature-based methods (e.g., Thornthwaite, 1948; Hamon, 1963). In this study, as we focused on the effects of spatial variability of soil texture on the relationship between mean and variance of soil moisture, we chose the simple temperature-based method (without considering vegetation control) (Hamon, 1963).

\[ PE = 0.0138D(\rho_{\text{sat}}(T_a)), \]  

where \( PE \) is daily potential evaporation rate in \text{cm/day}, \( D \) is day length in hour, and \( \rho_{\text{sat}} \) is the saturation absolute humidity at the mean daily temperature \( T_a \) in \text{g/m}^3 and given by

\[ \rho_{\text{sat}} = 0.622 \rho_a e_{\text{sat}}/P, \]  

where \( \rho_a \) is air density in \text{g/m}^3, \( P \) is the air pressure in \text{mb} and \( e_{\text{sat}} \) is the saturation vapor pressure in \text{mb}, which is computed as a function of air temperature

\[ e_{\text{sat}} = 6.11 \exp \left( \frac{17.3T_a}{T_a + 237.3} \right) \]  

In addition to evaporation, another dry-down factor is vertical drainage, which can be expressed as a
function of soil moisture following Clapp and Hornberger (1978)

\[
D = K_s \left( \frac{\theta}{\theta_s} \right)^{2b+3},
\]

(9)

where \( K_s \) is the soil saturated hydraulic conductivity. \( K_s, b, \theta_s, \) and \( \psi_s \) depend on the soil PSD. To understand the effects of spatial variability of soil texture on the relationship between mean and variance of soil moisture fields, we adopt empirical relationships, known as water retention curves, between soil PSD and soil hydraulic properties developed by Cosby et al. (1984).

\[
b = 2.91 + 0.159 \times \text{clay}\% \\
\psi_s = 10^{(1.88 - 0.0131 \times \text{sand}\%)} \text{(cm)} \\
K_s = 60.96 \times 10^{-0.84 + 0.0153 \times \text{sand}\%} \text{(cm/day)} \\
\theta_s = 48.9 - 0.126 \times \text{sand}\%
\]

(10)

The main reason for us to choose the Cosby et al. (1984) empirical relationships is that these relationships are simple, robust, and developed based on 1,448 soil samples.

After defining two dry-down factors, i.e., evaporation and drainage, now we can use the following differential equation to describe the soil moisture dry-down process starting from saturation

\[
z \frac{d \theta}{dt} = -E - D, \quad \theta(t = 0) = \theta_s,
\]

(11)

where \( z \) is soil thickness. If we know soil PSD at one site, we can obtain a time series of soil moisture at that site through solving Equation (11). For example, if \( \theta_A(t) \) and \( \theta_B(t) \) are soil moisture at sites A and B. The mean (\( \mu_\theta(t) \)) and variance (\( \sigma^2_\theta(t) \)) of soil moisture for sites A and B at time t are given by

\[
\mu_\theta(t) = \frac{\theta_A(t) + \theta_B(t)}{2},
\]

\[
\sigma^2_\theta(t) = \frac{[\theta_A(t) - \mu_\theta(t)]^2 + [\theta_B(t) - \mu_\theta(t)]^2}{2}
\]

(12)

Similarly, for a soil moisture field consisting of \( n \) measurement sites, we can first solve soil moisture at each site based on Equation (11), and then compute the mean and variance of soil moisture field as follows:

\[
\mu_\theta(t) = \frac{\sum_{i=1}^{n} \theta_i(t)}{n}, \quad \sigma^2_\theta(t) = \frac{\sum_{i=1}^{n} [\theta_i(t) - \mu_\theta(t)]^2}{n}
\]

(13)

CASE STUDIES

Idealized Case

Prior to applying our suggested method to explain the contradiction in the literature with regard to the relationship between mean and variance of soil moisture, we first used our method to study an idealized case, which is a soil moisture field with only two measurement sites: site A with a soil PSD of 20% clay and 50% sand, and site B with a soil PSD of 50% clay and 20% sand. The soil moisture sampling depth is 5 cm. To compute the potential evaporation rate, we set air temperature to be 20°C for both sites.

Solving Equation (11) for sites A and B, we obtain time series of soil moisture at each site. Applying Equation (12), we compute mean and variance of soil moisture. The plot of mean vs. variance of soil moisture shown in Figure 2 indicates that if mean soil moisture is greater than 24.5 (\% v/v), variance of soil moisture increases as mean soil moisture decreases. If mean soil moisture is less than 24.5 (\% v/v), the variance decreases as mean soil moisture decreases. As mean soil moisture is at 24.5 (\% v/v), the variance reaches its maximum [i.e., 42.3 (\% v/v)^2]. Computing the pressure head for each site as the variance is at

![Figure 2: The Plot of Mean Soil Moisture vs. Variance of Soil Moisture for an Idealized Case. The variance of soil moisture reaches its maximum when mean soil moisture is equal to 24.5 (\% v/v).]
its maximum, we find that both sites A and B are in stage II (as \( \psi_A = -1,653 \) cm and \( \psi_B = -6,182 \) cm are between \( -330 \) cm and \( -1,500 \) cm). To explain two different relationships between mean and variance obtained in this idealized case, let us consider a soil moisture dry-down process in each stage.

In stage I, as soil in both sites are wet, there is no water limit for evaporation and the evaporation rate is close to an atmospheric controlled potential rate that has a relatively small spatial variation. Therefore, evaporation can only reduce mean soil moisture, but not change variance of soil moisture. The other dry-down factor, drainage, is a function of soil hydraulic conductivity as shown in Equation (9). Because in this paper we focus on the effects of soil texture on the relationship between mean and variance of soil moisture fields, we may temporarily neglect the influences of soil structure (e.g., macropores, bulk density, etc.) on soil hydraulic conductivity. According to the linear relationships among soil PSD, \( K_n \), \( \theta_s \), and \( b \) given in Equation (10), we can find that there is a negative correlation between saturated soil moisture and \( K_n \). This negative correlation indicates that drainage will enhance variance of soil moisture, because relatively coarse textured soil (with a low saturated soil moisture) (i.e., Site A) dries faster than relatively fine textured soil (with a high saturated soil moisture) (i.e., Site B) under saturation or close-to-saturated condition. Therefore in stage I, variance of soil moisture increases (due to drainage) as mean soil moisture decreases (due to drainage and evaporation).

In stage II, the evaporation rate is no longer controlled by the atmospheric conditions only. The soil moisture condition plays an important role in controlling evaporation, i.e., the actual evaporation rate is proportional to soil moisture. In this stage, with decreasing soil moisture, the drainage decreases dramatically because drainage is proportional to the \((2b + 3)\) power of saturation \( (\theta/\theta_s) \) based on Equation (9). As soil moisture decreases to a threshold, the actual evaporation rate will surpass the drainage rate and the dominant drying factor will shift from drainage to evaporation. Therefore, the positive correlation between soil moisture and evaporation will reduce the spatial variability of soil moisture, and thus variance of soil moisture will decrease as mean soil moisture decreases.

In stage III, soil is so dry that the drainage is almost zero. Evaporation is the only factor to dry soil moisture. According to Equation (4), the actual evaporation rate is proportional to \( (\theta_2/\theta_1) \)

\[
E = PE \times (\theta_2/\theta_1) \tag{14}
\]

Substituting Equation (5) into Equation (14), we have

\[
E = PE \times (\psi_1/\psi_2)^{1/b} \tag{15}
\]

As \( \psi_1/\psi_2 < 1 \), \( E \) is positively correlated with \( b \), i.e., coarser soil, smaller \( b \), and smaller \( E \). Because soil moisture at coarse textured soil site (i.e., site A) is less than the soil moisture at fine textured soil site (i.e., site B), site A soil will dry more slowly due to a smaller \( E \) compared with site B soil. Therefore, in stage III evaporation will continue to reduce the spatial variability of soil moisture.

Overall, we can see that if mean soil moisture is greater than a threshold, soil moisture variance increases as mean soil moisture decreases. If mean soil moisture is less than the threshold, variance decreases as mean soil moisture decreases. Therefore, the dynamic range of observed mean soil moisture could mislead our understanding of the relationship between mean and variance of soil moisture, and create a contradiction in the literature with regard to this issue. In this study, we show three examples chosen from the literature. These three examples represent three different conclusions on the relationship between mean and variance of soil moisture.

**Owe et al.’s Case**

Owe et al. (1982) measured soil moisture during nine sampling events (1976-1978) at a test site in South Dakota in three surface horizons (0-2.5, 0-5, and 0-10 cm). The soils are loamy soils. For all soil moisture data collected in each horizon, they found that the variances of soil moisture were at a maximum as mean soil moisture between 15 \( (\% \ v/v) \) and 25 \( (\% \ v/v) \). The observed dynamic ranges of their observed mean soil moisture for 0-2.5, 0-5, and 0-10 cm are 1.8-30.5 \( (\% \ v/v) \), 2.6-30.3\( (\% \ v/v) \), and 4.6-30.5 \( (\% \ v/v) \), respectively. Because no soil PSD at each soil moisture sampling site was given in Owe et al. (1982), to explain the observed variance behavior by Owe et al. (1982), we chose two soil PSDs around the center of loamy soil in soil texture triangle map, i.e., Site 1 (45\% sand and 20\% clay), and Site 2 (40\% sand and 25\% clay). Applying the soil moisture dry-down equation to these sites, we computed soil moisture in horizons during a dry-down process from saturation to residual soil moisture. The plot of mean vs. variance is shown in Figure 3. All plots show that variances of soil moisture are at a maximum around 20 \( (\% \ v/v) \). Inside the dynamic ranges of observed mean soil moisture (as a shaded area shown in Figure 3) both increasing and decreasing of soil moisture variance with decreasing mean
soil moisture occur (Figure 3). According to Owe et al.’s observed data, we cannot conclude that soil moisture variance only increases or only decreases with decreasing of mean soil moisture.

**Famiglietti et al.’s Case**

Famiglietti et al. (1998) conducted a series field measurements of soil moisture in 0-5 cm along a hillslope transect. There were 21 sampling points. According to their measurements, they found that variance of soil moisture decreases with decreasing of mean soil moisture content. To explain such behavior, we read 21 soil PSDs from Figure 10 in Famiglietti et al. (1998). Applying soil moisture dry-down equation to these 21 sites, we computed soil moisture at each site during a dry-down process from saturation to residual soil moisture. The plot of mean vs. variance is shown in Figure 4. In the whole dynamic range of mean soil moisture, we can see that variance not only decreases with decreasing mean soil moisture, but also increases with decreasing mean soil moisture. If we only look at the relationship between variance and mean soil moisture in the dynamics range of Famiglietti et al.’s observed mean soil moisture (as a shaded area shown on Figure 4), we can find that the dominant pattern is the decreasing trend of variance with decreasing mean soil moisture. Between 25 (% v/v) and 35 (% v/v), our modeled variance increases with decreasing mean soil moisture (see Figure 4), but this pattern did not show in Famiglietti et al.’s Figure 3. Re-examining Famiglietti et al.’s Figure 3, we can find that between 25 (% v/v) and 35 (% v/v) three are three variances which are greater than 60 [(% v/v)²] and scatter of variance increases as mean soil moisture increases. We think that such scattering pattern and some extreme high variances are due to the spatial variability of topography, especially as soil is wet. Famiglietti et al.’s soil moisture data were collected along a hillslope with a slope of 0.17 m/m. As the effects of spatial variability of topography on the relationship between mean and variance of soil moisture are not considered in this study, our simulated variance does not show the same pattern as Famiglietti et al.’s as mean soil moisture is between 25 (% v/v) and 35 (% v/v).

**Hupet and Vanclooster’s Case**

Hupet and Vanclooster (2002) measured soil moisture content in a small maize cropped field located in Belgium. Soil moisture sampling depths are between 0 and 125 cm. Their data showed that variance of soil moisture increases with decreasing mean soil moisture. In Hupet and Vanclooster (2002), only one soil PSD in the surface layer (0-40 cm) was given, i.e., 6% sand and 12% clay. To explain variance behavior, we chose two PSDs, one is 6% sand and 12% clay, and the other PSD is 11% sand and 7% clay. We computed soil moisture in 0-20 cm for each site during a dry-down process. The plot of mean vs. variance is shown in Figure 5. Similar to other cases, variance increases with decreasing mean soil moisture if mean soil moisture is greater than 21 (% v/v), and decreases with decreasing mean soil moisture if mean
moisture is less than 21 (% v/v). Re-examining Figure 6 in Hupet and Vanclroost (2002), we can find that observed dynamic range of mean soil moisture by Hupet and Vanclroost is between 21 (% v/v) and 45 (% v/v). If we only look at the variance behavior inside Hupet and Vanclroost’s observed dynamic range of mean soil moisture (as a shaded area shown on Figure 5), we could conclude that variance increases with decreasing mean soil moisture, which is consistent with Hupet and Vanclroost’s observation.

CONCLUSIONS

In this paper, we focus on the effects of spatial variability of soil texture on the relationship between mean and variance of soil moisture during soil moisture dry-down processes. A nonlinear differential equation describing soil moisture dry down is proposed. In this equation, there are two terms, i.e., evaporation and drainage, controlling soil moisture dry down. We characterize the actual evaporation rate as a function of soil moisture and potential evaporation rate based on the concept of three stages of evaporation. The drainage term is based on Clapp and Hornberger (1978). After solving soil moisture dry-down equation during a single dry-down process at all soil moisture-sampling sites, we can calculate mean and variance of soil moisture fields at each time step. Our study shows that as mean soil moisture is greater than a threshold, variance increases with decreasing mean soil moisture. If mean soil moisture is less than the threshold, variance decreases with decreasing mean soil moisture. This threshold occurs in the stage II of evaporation and is between the field capacity and the wilting point. As the field capacity and the wilting point are functions of soil texture or soil PSD, the threshold of the relationship between mean and variance of soil moisture fields also depends on soil texture or soil PSD.

Through studying an idealized soil moisture field and three observed soil moisture fields chosen from Owe et al. (1982), Famiglietti et al. (1998), and Hupet and Vanclroost (2002), we demonstrated that the dynamic range of observed mean soil moisture could directly affect our observations of the relationship between mean and variance. For example, if the dynamic range of the observed mean soil moisture is between saturation and the threshold, we could think that variance increases with decreasing mean soil moisture. If the dynamic range of observed mean soil moisture is between the threshold and the residual soil moisture, we could conclude that variance of soil moisture decreases with decreasing mean soil moisture. If the observed soil moisture is in the whole dynamic range of soil moisture (i.e., between saturated soil moisture and residual soil moisture), we can find that soil moisture variance is maximum in the mid-range of mean soil moisture content.

As in this study we only focused on the effects of spatial variation of soil texture and neglected the differences in climate, vegetation, and topography, the conclusion drawn from this study may only be valid for small scales or the scales at which the spatial variations of climate, vegetation, and topography are insignificant. Nevertheless, we have demonstrated that the suggested method in this paper is simple and can be used to explain the apparent contradiction related to the relationship between mean and variance of soil moisture reported in the literature. More importantly, this study provides us the basis for the future study of the influences of vegetation, climate, and topography on the relationship between mean and variance of soil moisture fields.

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LITERATURE CITED
