Estimating daily root-zone soil moisture in snow-dominated regions using an empirical soil moisture diagnostic equation

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Abstract

Soil moisture in snow-dominated regions has many important applications including evapotranspiration estimation, flood forecasting, water resource and ecosystem services management, weather prediction and climate modeling, and quantification of denudation processes. A simple and robust empirical approach to estimate root-zone soil moisture in snow-dominated regions using a soil moisture diagnostic equation that incorporates snowfall and snowmelt processes is suggested and tested. A five-water-year approach to estimate root-zone soil moisture in snow-dominated regions using a soil moisture diagnostic equation (Su et al., 2013), and the accuracy of snow water equivalent and Peng, 2013; Kormos et al., 2014), early warm-season precipitation (Mahanama et al., 2012; Rosenberg et al., 2013; Abaza et al., Grant et al., 2004; Seyfried et al., 2005; Fassnacht et al., 2006; (5) forecasting water supply in groundwater systems (e.g., Barnett et al., 2005); (6) estimation of denudation processes including landslides (e.g., Ekinci et al., 2013), weathering, erosion, and mass movement (e.g., Leisenring and Moradkhani, 2012), such as dust storms and sand storms in China (Wang et al., 2010), and soil movement in periglacial regions (Matsuoka, 2005); (7) modeling ecosystem functions (e.g., Tague et al., 2009), such as greenhouse gas releases from boreal forest soils (e.g., Ullah et al., 2009) and burning in the New Zealand snow-tussock grasslands (Yeates and Lee, 1997).

Although soil moisture information has many vital applications in snow-dominated regions, unlike precipitation and air temperature, the direct in-situ observations of soil moisture are often inadequate in those regions. An alternative approach to produce accurate soil moisture information in snow-dominated regions is to estimate soil moisture using some numerical models. However, modeling soil moisture in snow-dominated regions is challenging because of the following reasons: (1) models need to differentiate between liquid and solid precipitation in the input data of precipitation, because rain water can directly infiltrate into soil, while

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Snowmelt
Snow water equivalent

1. Introduction

In snow-dominated regions, soil moisture information has many important applications, e.g., (1) estimation of evaporation and evapotranspiration (e.g., Robertson and Gazis, 2006; Christensen et al., 2008; Moyes and Bowling, 2013; Wang et al., 2013; Brown et al., 2014); (2) flood forecasting; without accurate soil moisture information, floods in snow-dominated regions cannot be predicted or modeled accurately (e.g., Koster et al., 2000; Grant et al., 2004; Seyfried et al., 2005; Fassnacht et al., 2006; Mahanama et al., 2012; Rosenberg et al., 2013; Abaza et al., 2014; Broxton et al., 2014; Nied et al., 2014; Salathe et al., 2014); (3) effects of soil moisture on the snowmelt process (e.g., Tague and Peng, 2013; Kormos et al., 2014), early warm-season precipitation (Su et al., 2013), and the accuracy of snow water equivalent measurements (e.g., Ouellette et al., 2013); (4) soil moisture impact on forest wildfire activity (e.g., Westerling et al., 2006); (5) forecasting water supply in groundwater systems (e.g., Barnett et al., 2005); (6) estimation of denudation processes including landslides (e.g., Ekinci et al., 2013), weathering, erosion, and mass movement (e.g., Leisenring and Moradkhani, 2012), such as dust storms and sand storms in China (Wang et al., 2010), and soil movement in periglacial regions (Matsuoka, 2005); (7) modeling ecosystem functions (e.g., Tague et al., 2009), such as greenhouse gas releases from boreal forest soils (e.g., Ullah et al., 2009) and burning in the New Zealand snow-tussock grasslands (Yeates and Lee, 1997).
snow is solid and accumulates on the ground; (2) estimates of snowmelt are required for modeling soil moisture, while snowmelt usually is not directly measured but rather is estimated from observed snow water equivalent (SWE) if SWE is measured; (3) lack of SWE measurements adds another difficulty in modeling soil moisture in snow-dominated regions, because models also have to simulate SWE; and (4) the freeze-thaw cycle occurring in high latitude or high altitude soils makes soil moisture modeling more challenging.

There are many sophisticated numerical models published in the literature that can capture soil moisture dynamics well in snow-dominated regions, e.g., GISS GCM (Lynch-Stieglitz, 1994), VIC (Liang et al., 1994, 1996), CLSM (Koster et al., 2000), CLM (e.g., Yang and Niu, 2003; Niu and Yang, 2003), and others. However, applying these sophisticated models to simulate soil moisture in snow-dominated regions is a difficult task, requiring a large amount of input data and parameters to run the models and to validate the simulated results. Furthermore, sometimes soil moisture data in snow-dominated regions are not available or difficult to obtain. To solve the problem of the scarcity of soil moisture data in snow-dominated regions, to provide initial and boundary soil moisture conditions for running some sophisticated numerical models, and to produce temporal coverage of spatially distributed soil moisture fields for verifying those models, this study aims to develop a simple and robust empirical approach to estimate daily root-zone soil moisture in snow-dominated regions using the soil moisture diagnostic equation proposed by Pan et al. (2003), Pan (2012), and Pan et al. (2015). In addition to producing soil moisture data for calibrating and validating numerical models, the method proposed in this study can also provide an efficient way to generate soil moisture data for calibrating satellite sensors and remote sensing algorithms to produce remotely sensed soil moisture products, e.g., Advanced Microwave Scanning Radiometer – Earth Observing System Sensor (AMSR-E) on the NASA Aqua satellite (Coopersmith et al., 2015a), the NASA Soil Moisture Active and Passive mission (SMAP) at the global scale (e.g., Cheema et al., 2011), as well as other applications such as irrigation scheduling, agricultural yield estimation, wildfire prediction and prevention, ecosystem management, and water resources management (e.g., Coopersmith et al., 2014, 2015b).

Pan et al. (2003) and Pan (2012) derived a daily soil moisture diagnostic equation (SMDE) based on a linear stochastic differential equation suggested by Entekhabi and Rodriguez-Iturbe (1994). As shown in Pan et al. (2003), Pan (2012) and Pan et al. (2015), the soil moisture diagnostic equation is a robust empirical approach to estimate soil moisture with four advantages, i.e., (1) no initial soil moisture is needed; (2) errors in the estimated soil moisture are not cumulative; (3) thus no recalculation is needed; (4) soil moisture can be estimated in a wide range of thicknesses of the soil column. With respect to the last point, for example, Pan et al. (2015) showed that applicability of the SMDE is not just limited to the surface soil moisture, the root zone soil moisture can also be estimated or predicted using the SMDE. In Pan et al. (2015), the SMDE has been successfully applied to estimate soil moisture in 0–10 cm, 0–20 cm, 0–50 cm, and 0–100 cm soil columns.

The main objective of this study is to test if the soil moisture diagnostic equation can be used to accurately estimate daily root-zone soil moisture in snow-dominated regions through including snowfall and snowmelt processes in the soil moisture diagnostic equation. The second goal of this study is to demonstrate if the sinusoidal soil moisture loss function also works well in snow-dominated regions to represent soil water loss due to evapotranspiration and drainage, as shown in Pan (2012). The arrangement of this paper is as follows. Section 2 describes the soil moisture diagnostic equation including snowfall and snowmelt processes, and the application of the soil moisture diagnostic equation to estimate soil moisture. Section 3 contains information about 12 SNOTEL sites across Utah used in this study for demonstrating the ability of the soil moisture diagnostic equation in estimating root-zone soil moisture in snow-dominated regions. Section 4 presents results and discussion. Conclusions are given in Section 5.

2. Method

2.1. Derivation of a daily soil moisture diagnostic equation with snow processes

Pan et al. (2003) first derived the daily soil moisture diagnostic equation (SMDE) based on a linear stochastic differential equation (Entekhabi and Rodriguez-Iturbe, 1994). Using the same approach, we can derive a similar soil moisture diagnostic equation including snowfall and snowmelt processes based on a simplified soil moisture dynamic equation given as follows:

\[ \frac{dz}{dt} = -\eta z + \gamma W \]  \hspace{1cm} (1)

where \( z \) is the thickness of a soil column (from land surface down to depth \( z_1 \)), \( \theta \) is soil moisture of the soil column, \(-\eta\) is the loss of soil moisture, \( \eta \) is the loss coefficient, \( W \) is the liquid water input rate including liquid precipitation and snowmelt, and \( \gamma \) is the infiltration coefficient representing the ratio of infiltration to liquid water input. Eq. (1) states that the soil moisture time-change rate is equal to infiltration minus soil moisture loss. Vertical drainage and evaporation or evapotranspiration (ET) are two principal processes controlling soil water loss. Rearranging terms in Eq. (1) yields:

\[ \frac{zd\theta}{C\eta} + \Gamma \theta W = dt \]  \hspace{1cm} (2)

For a time series of soil moisture at a point illustrated in Fig. 1, Eq. (2) can be integrated between time \( t_2 \) and \( t_1 \) as follows:

\[ \int_{t_2}^{t_1} -\frac{zd\theta}{C\eta} + \Gamma \theta W = \int_{t_2}^{t_1} dt \]  \hspace{1cm} (3)

where \( t_2 < t_1 \). For a time step less than or equal to one day the loss coefficient and the infiltration coefficient can be assumed to be constants between time \( t_2 \) and \( t_1 \). \( W \) in Eq. (3) is the observed or estimated liquid water input between time \( t_1 \) and \( t_2 \) and independent of soil moisture. With the above assumptions, Eq. (3) becomes:

\[ \frac{z}{C' \eta_1} \ln \left[ \frac{\theta_1 - \gamma W_1 / \eta_1}{\theta_2 - \gamma W_1 / \eta_1} \right] = t_1 - t_2 \]  \hspace{1cm} (4)

where \( \eta_1 \) and \( W_1 \) are the loss coefficient and total liquid water input between time \( t_1 \) and \( t_2 \), respectively. Rearranging Eq. (4) yields:

\[ \theta_1 = \theta_2 e^{\frac{-\gamma W_1}{\eta_1}} + \frac{\gamma W_1}{\eta_1} \left[ 1 - e^{\frac{-\gamma W_1}{\eta_1} (t_2 - t_1)} \right] \]  \hspace{1cm} (5)

For a daily time step (i.e., \( t_1-t_2 = 1 \) day), Eq. (5) can be written as:

\[ \theta_1 = \theta_2 e^{\frac{-\gamma W_1}{\eta_1}} + \frac{\gamma W_1}{\eta_1} \left( 1 - e^{\frac{-\gamma W_1}{\eta_1}} \right) \]  \hspace{1cm} (6-1)

where \( \eta_1, W_1, \theta_1, \theta_2 \) are daily soil moisture loss coefficient, liquid water input of day 1, soil moisture of day 1, and soil moisture of day 2 (which is one day before day 1), respectively. Similarly, we can have soil moisture of day 2 \( (\theta_2) \) as a function of \( \eta_2, W_2, \) and \( \theta_2 \):

\[ \theta_2 = \theta_2 e^{\frac{-\gamma W_2}{\eta_2}} + \frac{\gamma W_2}{\eta_2} \left( 1 - e^{\frac{-\gamma W_2}{\eta_2}} \right) \]  \hspace{1cm} (6-2)

and soil moisture of day \( n-1 \) \( (\theta_{n-1}) \) as a function of \( \eta_{n-1}, W_{n-1}, \) and \( \theta_{n-1} \):
that the B value approaches a stable value as n increases (Pan et al., 2003). Since, the infiltration coefficient declines as B increases, an exponential curve is the best fit of soil moisture as a function of the B value (Pan et al., 2003). Therefore, a general form of soil moisture as a function of the B value should be:

$$\theta = \theta_{re} + (\theta_{re} - \theta_{se})(1 - e^{-B})$$

(10)

where $$\theta_{re}$$ and $$\phi_c$$ are effective residual soil moisture and effective saturated soil moisture, respectively, $$c_4$$ is an empirical constant related to soil hydraulic properties. Eq. (10) is called the daily soil moisture diagnostic equation. If we set $$z$$ in Eq. (1) to be the thickness of the root zone, and let the loss coefficient represent the effective soil moisture loss from the entire root zone, Eq. (10) becomes the root-zone soil moisture diagnostic equation.

To use the soil moisture diagnostic equation to estimate root-zone soil moisture in snow-dominated regions, we also need to estimate the liquid water input rate. During the snowfall process, snow accumulates on the ground and no infiltration happens if snowmelt is insignificant, or the liquid water input rate is zero. During the snowmelt process, in addition to liquid precipitation (i.e., rainfall), snowmelt will also add liquid water into soils. Therefore, the liquid water input consists of liquid precipitation (i.e., rainfall) and snowmelt, and is estimated as follows:

$$W_i = LP_i + SM_i$$

(11)

where $$W_i$$ is the liquid water input rate, $$LP_i$$ is liquid precipitation rate, and $$SM_i$$ is snowmelt of day i, respectively. In this study, rain and snow are separated based on the commonly used method (e.g., Motyoma, 1990; Lynch-Stieglitz, 1994; Kienzle, 2008), i.e., if air temperature is greater than 0°C it is rain, otherwise, it is snow:

\[\text{If } T_i > 0 \degree \text{C, } LP_i = P_i, \text{SNOW}_i = 0 \]
\[\text{If } T_i \leq 0 \degree \text{C, } LP_i = 0, \text{SNOW}_i = P_i \]

(12)

where $$T_i$$ is daily mean air temperature, and $$\text{SNOW}_i$$ is the liquid equivalent snowfall rate of day i. Snowmelt of day i ($$\text{SM}_i$$) is estimated based on the measured snow water equivalents of day i-1 ($$\text{SWE}_{i-1}$$) and day i ($$\text{SWE}_i$$) as follows:

$$\text{SM}_i = \text{SWE}_{i-1} + \text{SNOW}_i - \text{SWE}_i$$

(13)

2.2. The soil moisture loss coefficient

To apply the soil moisture diagnostic equation, we must first determine the soil moisture loss coefficient that represents the ratio between soil moisture loss due to drainage and evapotranspiration (over bare ground) or evapotranspiration (ET) over vegetated land surface and soil moisture. In addition to soil moisture, drainage is controlled by soil hydraulic properties and ET is related to the potential evapotranspiration (PET), therefore, the soil moisture loss coefficient depends on both soil hydraulic properties (controlling drainage) and the PET (controlling the actual ET rate). Soil hydraulic properties can be assumed as time-invariant variables and the PET varies with time as atmospheric conditions change temporarily. Thus the loss coefficient can be approximated by a sum of one constant that is related to soil hydraulic properties, and a function of time that represents the PET.

The potential evapotranspiration (PET) is also referred to as the atmospheric demand evapotranspiration and is controlled by the
atmospheric conditions, e.g., evaporation over bare soils and transpiration over vegetation are driven by solar radiation. Air temperature and relative humidity control the water vapor gradient between the atmosphere and the land surface. Wind speed influences the exchanges of water vapor and heat between the land surface and the atmosphere. The canopy structure also impacts the vertical profiles of air temperature, water vapor and wind speed, and in turn affects the water and heat fluxes between the land surface and the atmosphere. Since all these aforementioned atmospheric conditions (e.g., daily mean values of solar radiation, air temperature, and relative humidity at a location) can be approximated by functions of the Day of Year (DOY), Pan (2012) and Pan et al. (2015) used a sinusoidal wave function of DOY to represent the daily soil moisture loss coefficient \( c_i \) (which depends on soil hydraulic properties and the PET rate) as follows:

\[
\eta_i = c_1 + c_2 \sin\left(\frac{2\pi (DOY_i + c_3)}{365}\right) \tag{14}
\]

where \( \eta_i \) is the loss coefficient of day \( i \), DOY\(_i\) is the DOY of day \( i \), and \( c_1, c_2, \) and \( c_3 \) are constants, hereafter referred as the loss coefficient parameters. The root mean square errors (RMSEs) in the estimated surface soil moistures in Pan (2012) and the estimated root-zone soil moistures in Pan et al. (2015) are all less than 5%V/V. This indicates that a sinusoidal wave function of DOY represents the daily soil moisture loss coefficient well, although such a sinusoidal wave function could smooth out the high frequency soil moisture variations due to evapotranspiration and drainage. Therefore, in this study, Eq. (14) is also used to represent the root-zone soil moisture daily loss coefficient in snow-dominated regions.

2.3. Determination of parameters in the soil moisture diagnostic equation

There are six parameters associated with the soil moisture diagnostic equation approach, i.e., \( c_1, c_2, c_3 \) in the soil moisture loss function as shown in Eq. (14), and \( \theta_{rs}, \theta_r, \) and \( \theta_c \) in the soil moisture diagnostic equation (Eq. (10)). In Pan (2012) and Pan et al. (2015), these six parameters are determined in two steps. The first step is to search for the optimal values of \( c_1, c_2, \) and \( c_3 \) by maximizing the correlation coefficient between the observed soil moisture and the calculated B value (i.e., the best fit between the observed soil moisture and the B value). The second step is to inversely estimate \( \theta_r, \theta_c \) and \( \theta_c \) by fitting the scatter plot of the observed soil moisture versus the calculated B value based on the optimal values of \( c_1, c_2, \) and \( c_3 \) in the soil moisture loss function determined in the first step.

Pan (2012) and Pan et al. (2015) successfully applied the Monte Carlo method to determine the three parameters in the soil moisture loss function. There is another method, known as the Genetic Algorithm (GA), which has also been utilized to determine the optimal parameters in the soil moisture loss function by Cooper et al. (2014, 2015a, 2015b). Since the GA method is considered as a guided random search method, it could be more efficient than the Monte Carlo method (Gallagher et al., 1991) in terms of computational time, especially when the soil moisture diagnostic equation is applied to estimate hourly soil moisture. However, in this study we found that it took about less than 30 min of CPU time of a 2.3 GHz processor for determining the three parameters in the soil moisture loss function based on 1086 data points at each site using the Monte Carlo method. Given that the primary goal of this study is to demonstrate the feasibility of the suggested method for estimating root-zone soil moisture in snow-dominated regions, rather than to focus on the computational efficiency associated with the soil moisture diagnostic equation approach, in this study we still used the simple and feasible Monte Carlo search method to determine the three parameters in the soil moisture loss function.

To conduct a Monte Carlo search of the optimal parameters, we first need to set the parameter space. In the soil moisture loss function as shown Eq. (14), \( c_1 \) stands for the mean value of the loss coefficient, and \( c_3 \) is the magnitude of the loss coefficient variation, and \( c_2 \) is the phase shift of the sinusoidal wave. Because the loss coefficient cannot be negative, \( c_2 \) must be less than or equal to \( c_1 \). Both \( c_1 \) and \( c_2 \) are expressed in the same units as precipitation (i.e., length/day, because a daily time step is used in this study) and \( c_3 \) is expressed in the DOY (between 0 and 365). Pan et al. (2015) set the maximum \( c_1 \) to be 4 cm/day for arid and semi-arid regions according to the map of mean annual pan evaporation for the contiguous United States constructed by Farnsworth and Thompson (1982). In the mid- and high latitude snow-dominated regions, the potential evapotranspiration rate (PET) is much less than that in low and mid-latitude arid and semi-arid regions, and thus the dynamic range of 0–4 cm/day might also be sufficient for \( c_1 \). The searching domain (or parameter space) of the parameters in the soil moisture loss function is given as follows:

\[
\text{parameter space} = \{0 < c_1 < 4 \text{ cm/day}; 0 < c_2 < c_1; 0 < c_3 < 365\} \tag{15}
\]

However, if the determined optimal \( c_1 \) values are too close to 4.0 cm/day, we need to increase the dynamic range for \( c_1 \) and redo the Monte Carlo search, because the soil moisture loss coefficient is not only dependent on PET but also soil saturated hydraulic conductivity. This might happen in forest soils where macropores in soils can greatly increase soil saturated hydraulic conductivities.

3. Study sites and data

The Snow Telemetry (SNOTEL) is an extensive and automatic system installed, operated and maintained by the United States Department of Agriculture (USDA) Natural Resources Conservation Service (NRCS). In addition to measuring snowpack and other meteorological components (e.g., air temperature, precipitation, wind speed, relative humidity, solar radiation, etc.), some SNOTEL sites also take soil moisture measurements at depths of 5 cm, 20 cm and 50 cm twice per day (12:00 and 0:00 location time) using a dielectric constant measuring device with an average accuracy of ±3.0% (V/V) (Schaefver et al., 2007). Daily mean soil moisture at each depth is calculated as the arithmetic mean of these two measurements. The soil moisture data collected at all SNOTEL sites are soil moisture point measurements at different depths, while the soil moisture diagnostic equation is generally applied to estimate soil moisture in a soil column from the soil surface down to a certain depth. Therefore, the soil moisture point measurements at three different depths are used to compute mean soil moistures in three soil columns, i.e., 0–5 cm, 0–20 cm, and 0–50 cm, as follows:

\[
\begin{align*}
\theta_{0-5cm} &= \frac{5 \text{ cm } \times \theta_{5cm} + 15 \text{ cm } \times (\theta_{15cm} + \theta_{20cm})}{20 \text{ cm}} \\
\theta_{0-20cm} &= \frac{5 \text{ cm } \times \theta_{5cm} + 15 \text{ cm } \times (\theta_{15cm} + \theta_{20cm})}{2} + 30 \text{ cm } \times (\theta_{20cm} + \theta_{50cm})/2 \\
\theta_{0-50cm} &= 50 \text{ cm }
\end{align*}
\]

(16)

where \( \theta_{0-5cm}, \theta_{0-20cm}, \) and \( \theta_{0-50cm} \) are mean soil moistures in 0–5 cm, 0–20 cm, and 0–50 cm soil columns, respectively; and \( \theta_{5cm}, \theta_{15cm}, \theta_{20cm}, \) and \( \theta_{50cm} \) are measured soil moistures at depths of 5 cm, 20 cm, and 50 cm, respectively.

To test the ability of the soil moisture diagnostic equation to estimate root zone soil moisture in snow-dominated regions, we randomly selected 12 SNOTEL sites from all SNOTEL sites in Utah...
Locations, elevations, soil textures and land cover types of 12 SNOTEL sites in Utah.

<table>
<thead>
<tr>
<th>Site ID</th>
<th>Site name</th>
<th>Lat.</th>
<th>Lon.</th>
<th>Ele. (m)</th>
<th>Soil texture</th>
<th>Land-cover</th>
</tr>
</thead>
<tbody>
<tr>
<td>UT333</td>
<td>Ben Lomond Trail</td>
<td>41.38’N</td>
<td>111.92’W</td>
<td>1777</td>
<td>Loamy-skeletal</td>
<td>Forest</td>
</tr>
<tr>
<td>UT366</td>
<td>Brighton</td>
<td>40.6’N</td>
<td>111.583’W</td>
<td>2667</td>
<td>Loamy-skeletal</td>
<td>Tree/grass</td>
</tr>
<tr>
<td>UT528</td>
<td>Hole-in-Rock</td>
<td>40.917’N</td>
<td>110.183’W</td>
<td>2789</td>
<td>Loamy-skeletal</td>
<td>Tree</td>
</tr>
<tr>
<td>UT533</td>
<td>Horse Ridge</td>
<td>41.317’N</td>
<td>111.45’W</td>
<td>2487</td>
<td>Loamy-skeletal</td>
<td>Grass</td>
</tr>
<tr>
<td>UT582</td>
<td>Little Bear</td>
<td>41.4’N</td>
<td>111.833’W</td>
<td>1995</td>
<td>Clay</td>
<td>Shrub</td>
</tr>
<tr>
<td>UT884</td>
<td>Parley’s Summit</td>
<td>40.767’N</td>
<td>111.633’W</td>
<td>2286</td>
<td>Fine montmorillonitic</td>
<td>Tree/grass</td>
</tr>
<tr>
<td>UT686</td>
<td>Payson R.S.</td>
<td>39.933’N</td>
<td>111.633’W</td>
<td>2459</td>
<td>Loamy-skeletal</td>
<td>Grass</td>
</tr>
<tr>
<td>UT553</td>
<td>Webster Flat</td>
<td>37.583’N</td>
<td>112.9’W</td>
<td>2805</td>
<td>Fine montmorillonitic</td>
<td>Grass/tree</td>
</tr>
<tr>
<td>UT896</td>
<td>Hardscrabble</td>
<td>40.867’N</td>
<td>111.717’W</td>
<td>2210</td>
<td>Loamy-skeletal</td>
<td>Grass/tree</td>
</tr>
<tr>
<td>UT972</td>
<td>Louis Meadow</td>
<td>40.833’N</td>
<td>111.767’W</td>
<td>2042</td>
<td>Fine montmorillonitic</td>
<td>Shrub</td>
</tr>
<tr>
<td>UT1654</td>
<td>Farmington Lower</td>
<td>40.967’N</td>
<td>111.817’W</td>
<td>2066</td>
<td>Loamy-skeletal</td>
<td>Forest</td>
</tr>
<tr>
<td>UT1113</td>
<td>Tony Grove RS</td>
<td>41.883’N</td>
<td>111.567’W</td>
<td>1930</td>
<td>Loamy-skeletal</td>
<td>Grass</td>
</tr>
</tbody>
</table>

Table 1

Locations, elevations, soil textures and land cover types of 12 SNOTEL sites in Utah.

4. Results and discussion

4.1. Determination of parameters in the soil moisture diagnostic equation

As introduced in Section 2.1, we need to replace the precipitation rate in the soil moisture diagnostic equation (SMDE) by the liquid water input rate for snow-dominated regions. To demonstrate this, we conducted a set of two tests in each soil column at each site, i.e., with and without replacing the precipitation rate in the SMDE by the liquid water input rate, and compared the results. For these 12 SNOTEL sites we found that a 4-month window size was sufficiently large enough to calculate the B value. We first used the Monte Carlo search method to determine the optimal parameters in the soil moisture loss function for each of three soil columns (i.e., 0–5 cm, 0–20 cm, and 0–50 cm) at 12 SNOTEL sites with two treatments (i.e., with and without replacing the precipitation rate in the SMDE by the liquid water input rate). The determined optimal parameters \(c_1, c_2,\) and \(c_3\) and the associated correlation coefficients \(r_B\) between the observed soil moisture in the PEP (i.e., 10/1/2010–9/30/2013) and the computed B values are listed in Table 2. All \(r_B\) values are greater than 0.65, no matter whether the snowfall and snowmelt processes are considered or not considered in the soil moisture diagnostic equation, which indicates that the sinusoidal wave function of DOY also can be used to approximate the daily soil moisture loss in snow-dominated regions, although some high frequency variations in daily soil moisture might be smoothed out by such approximation. According to Table 2, including the snowfall and snowmelt processes in the soil moisture diagnostic equation increased the correlation coefficients \(r_B\) in 27 cases, produced the same correlation coefficients in 2 cases (i.e., 0–5 cm at UT582 and UT686), and decreased the correlation coefficients in 7 cases (i.e., 0–5 cm, 0–20 cm, and 0–50 cm at UT582 and UT853, and 0–50 cm at UT884), corresponding to 75%, 5.5%, and 19.5% of 36 total cases, respectively. The average correlation coefficient for the three soil columns (i.e., 0–5 cm, 0–20 cm, and 0–50 cm) at each site is also listed in Table 2. Among 12 SNOTEL sites, 9 sites (75%) have higher average correlation coefficients; one site (8.3%) has the same average coefficient; and two sites (16.7%) have lower average coefficients after snowfall and snowmelt processes are included in the SMDE compared to those after snowfall and snowmelt processes are not considered in the SMDE. This indicates that considering snowfall and snowmelt processes and replacing the precipitation rate in the SMDE by the liquid water input rate generally can improve the correlation between soil moisture and the B value. The decline of the correlation coefficients at two sites was probably due to the uncertainty associated with using daily mean temperature of 0 °C as the single threshold to separate snow and rain.

To determine the parameters in the soil moisture diagnostic equation (i.e., Eq. (10)), in each of three soil columns at each site, we used the determined optimal parameters of the soil moisture loss function to compute the B values in the parameter estimation period (i.e., PEP) and produced the scatter plot of the observed soil moisture data versus the computed B values. Finally we fitted the scatter plot by Eq. (10) for determining the three parameters in Eq. (10), i.e., \(\theta_a\) and \(\phi_a\) are effective residual soil moisture and effective saturated soil moisture, respectively, and \(c_4\) is an empirical constant related to soil hydraulic properties. There are three methods that can be used to determine these three parameters. The first method (M1) is to directly determine these parameters using the least-squares curve fitting method by setting the lower limits of all these three parameters to be zero, i.e., all parameters are positive. The second method (M2) is to set \(\theta_a\) and \(\phi_a\) to be the observed minimum and maximum soil moisture during the parameter estimation period (PEP), respectively, and use the least-squares curve fitting method to determine the optimal value of \(c_4\). If soil texture or soil particle size distribution (PSD) is known at the study site, we can use some published empirical formulas (e.g., Rawls et al., 1982; Saxton et al., 1986) to estimate \(\theta_a\) and \(\phi_a\) based on soil texture or PSD, and then the least-squares curve fitting method is applied to determine \(c_4\). However, this third method (M3) is not as common...
as the first and second methods because of uncertainty in the published empirical formulas and unavailability of soil texture or soil PSD. Since no soil pedon reports at the 12 study sites are available, only the first and second methods can be used in this study.

Unlike the first method, in the second method when we use the observed minimum and maximum soil moistures during the parameter estimation period (PEP) to approximate \( \theta_r \) and \( \theta_e \), the dynamic range of the observed soil moisture in the PEP directly affects the accuracy of the estimated \( \theta_r \) and \( \theta_e \). For example, if the PEP is too short to cover the whole possible dynamic range of soil moisture, using the second method to determine \( \theta_r \) and \( \theta_e \) would result in an overestimated \( \theta_r \) and an underestimated \( \theta_e \). In this study, the length of the PEP is three water years that cover three cycles of the driest (late summer) and wettest (late spring and early summer as snow melts) seasons, therefore the second method should work well. For the first method, the dynamic range of the observed soil moisture is also important, because if the length of the PEP is too short and the observed minimum and maximum soil moistures may be not close to the actual effective residual soil moisture and effective saturated soil moisture, respectively. On the other hand, the least-squares curve

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**Fig. 2.** Daily temperature, daily snow water equivalent, daily precipitation, daily water input rate (rain + estimated snowmelt), and soil moistures in three soil columns.
fitting with three parameters to be determined (i.e., with three degrees of freedom) could lead to an overestimated residual soil moisture and an underestimated saturated soil moisture, because the majority of data points usually falls within the middle range of soil moisture, rather than the lower or higher soil moisture ranges, and each data point is given the same weight for the curve fitting. To compare difference between the first and second methods, three parameters in the SMDE (i.e., \(h_{re}\), \(e\), and \(c_4\)) determined by the first and second methods as the snow processes are considered in the SMDE are listed in Table 3. According to Table 3, differences between the effective residual soil moistures (\(h_{re}\)) determined by M1 and M2 methods are less than 1.0 (%V/V) in 18 out of 36 cases (50%), and greater than 1.0 (%V/V) but less than 5.1 (%V/V) in the other 18 cases. However, in 32 out of 36 cases (89%), the M2 determined effective saturated soil moistures (\(e\)) are greater than those determined by M1 method, and the largest difference is 14.0 (%V/V). Only in 4 out of 36 cases (11%), the M2 determined saturated soil moistures are less or close to those determined by the M1 method. These results indicate that the M1 method tends to underestimate \(e\), and the differences in the M1 and M2 determined effective residual soil moistures are not significant.

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Considering that the effective residual soil moisture and the effective saturated soil moisture are the lowest and highest limits of the soil moisture state in a particular soil column at a particular site, respectively, we can combine the M1 and M2 determined \( \theta_e \) and \( \theta_s \), and set \( \theta_i \) to be the minimum value of the M1 and M2 determined \( \theta_i \), and \( \theta_s \) to be the maximum value of the M1 and M2 determined \( \theta_i \). Then the parameter \( c_4 \) is determined using the least-squares curve fitting with the newly determined \( \theta_i \) and \( \theta_s \). We call this approach the M12 method. The M12 results are also listed in Table 3. According to Table 3, the M1 method produced the smallest root mean square errors (RMSEs) in the estimated soil moistures and the highest correlation coefficients (r) among the three methods (i.e., M1, M2, and M12). Naturally this gives rise to a question: what is the benefit of using the M12 method? To address this question, we plotted the scatter plots of the computed B values versus the observed soil moistures in three soil columns (i.e., 0–5 cm, 0–20 cm, and 0–50 cm) at UT333 in Fig. 3. The best fit curves of the scatter plots using the M1, M2, and M12 determined parameters are also shown in Fig. 3. According to Fig. 3, we can find that the differences between M2 and M12 best-fit curves for three soil columns are all insignificant and such insignificant differences are due to small differences in \( \theta_i \) (i.e., 1.3% V/V for 0–5 cm, 2.25% V/V for 0–20 cm, and 3.5% V/V for 0–50 cm). However, there is a significant difference between M1 and M2 and between M1 and M2 best-fit curves, because a smaller \( \theta_s \) makes the M1 best-fit curve flat at a smaller B value than the M2 or M12 best-fit curves in each soil column. The time series plots of the observed and estimated soil moistures based on M1, M2, and M12 determined parameters in three soil columns at UT333 plotted in Fig. 4 clearly show that using the M1 determined parameters in the SMDE in soil moisture estimation could result in under-predicting high soil moistures and over-predicting low soil moistures. However, the underestimation of high soil moisture and overestimation of low soil moisture are eliminated in the M12 method. Although the RMSEs of the estimated soil moistures using the M12 determined parameters are slightly higher than those of M1 or M2, we think the M12 method is the best and should be used to determine the three parameters in the soil moisture diagnostic equation because it can better capture high and low soil moisture dynamics.

4.2. Estimation of soil moisture using the SMDE

It is straightforward to estimate soil moisture using the SMDE after three parameters (i.e., \( c_1 \), \( c_2 \), and \( c_3 \)) in the sinusoidal soil moisture loss function (i.e., Eq. (14)) and three parameters (i.e., \( \theta_i \), \( \theta_e \), and \( c_4 \)) in the SMDE (i.e., Eq. (10)) are determined, which consists of four steps: (1) determine \( c_1 \), \( c_2 \), and \( c_3 \) using the Monte Carlo search method; (2) compute the B values; (3) determine \( \theta_i \), \( \theta_e \), and \( c_4 \) using the M12 method (see Section 4.1); and (4) calculate soil moistures using the computed B values and the SMDE. In each of three soil columns at each of 12 SNOTEL sites, we first calculated soil moistures in both parameter estimation period (PEP) and model testing-testing period (MTP) as the snow and snowmelt processes were considered in the SMDE and then plotted the time series plots of the observed and estimated soil moistures in Fig. 5. To evaluate the errors in the estimated soil moisture and effectiveness of the soil moisture diagnostic equation approach proposed in this study, we computed the root mean square errors (RMSEs) and correlation coefficients between the observed and estimated soil moistures, which are listed in Table 4. The RMSEs ranged between 2.64 (3% V/V) and 6.23 (3% V/V), and the correlation coefficients varied between 0.75 and 0.95. The overall average RMSE was 4.28 (3% V/V) and overall average correlation coefficient was 0.89. In addition to RMSE and correlation coefficient, the Nash-Sutcliffe model efficiency coefficient \( E_c \) was also computed for evaluating the estimated soil moistures, which is defined as follows:

\[
E_c = 1 - \frac{\sum_{i=1}^{n} (\theta_{oi} - \theta_{ei})^2}{\sum_{i=1}^{n} (\theta_{oi} - \overline{\theta})^2}
\]  

(17)
The Ec varied between 0.24 and 0.90, and the overall average Ec was 0.72. To assess the impact of the snow and snowmelt processes on soil moisture, we also computed soil moistures without considering the snow and snowmelt processes.

Table 3

Determined optimal parameters in the soil moisture diagnostic equation and the associated root mean square (RMSE) and correlation coefficient (r) between the observed and estimated soil moistures.

<table>
<thead>
<tr>
<th>Site ID</th>
<th>0-5 cm</th>
<th>0-20 cm</th>
<th>0-50 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>UT1113</td>
<td>M1: 0.0 35.2 0.87 3.93</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>UT1113</td>
<td>M2: 0.0 40.9 0.42 5.19</td>
<td>0.8</td>
<td>1.6</td>
</tr>
<tr>
<td>UT1113</td>
<td>M12: 9.3 38.7 1.71 4.87</td>
<td>0.6</td>
<td>1.2</td>
</tr>
</tbody>
</table>

M2: set \( h_0 \) and \( \Phi_0 \) to be the observed minimum and maximum soil moistures during the parameter estimation period (PEP), \( c_4 \) is the single parameter to be estimated by curve fitting.

M1: \( h_0 \) and \( \Phi_0 \) are estimated by curve fitting.

M12: set \( h_0 \) to be the minimum value of M2 estimated \( h_0 \) and M1 estimated \( h_0 \), set \( \Phi_0 \) to be the maximum value of M2 estimated \( \Phi_0 \) and M1 estimated \( \Phi_0 \), \( c_4 \) is the single parameter to be estimated by curve fitting.

where Ec is the Nash-Sutcliffe model efficiency coefficient, \( h_0 \) and \( \Phi_0 \) are observed and estimated soil moistures, respectively, and \( \bar{M} \) is mean observed soil moisture. The Ec varied between 0.24 and 0.90, and the overall average Ec was 0.72.

To assess the impact of the snow and snowmelt processes on the SMDE-based soil moisture estimation, we also computed soil moistures without considering the snow and snowmelt processes in the SMDE. The simulated soil moistures are also plotted in Fig. 5. The RMSEs, correlation coefficients, and the Ecs are also listed in Table 4. According to Table 4, the RMSEs ranged between 2.35 (%V/V) and 7.34 (%V/V), the correlation coefficients varied between 0.50 and 0.94, and the Ecs were in the range of 0.22 and 0.87. The overall average RMSE was 4.97 (%V/V), the average correlation coefficient was 0.82, and the average Ec was 0.64. These...
Fig. 4. Observed and estimated soil moisture in 0–5 cm, 0–20 cm, and 0–50 cm soil columns at UT333 using the parameters in the SMDE estimated by the M2, M1, and M12 methods.

Fig. 5. Observed and estimated soil moistures at 12 SNOTEL sites.

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results indicate that ignoring the snow/snowmelt processes in the SMDE generally produced a larger RMSE, a lower correlation coefficient, and a lower Nash-Sutcliffe coefficient Ec, because 83.3%, 94.4% and 84.7% of 72 cases show that the inclusion of the snow/snowmelt processes in the SMDE yielded a smaller RMSE, a higher correlation coefficient, and a higher Ec than in the models that ignored the snow/snowmelt processes. Both the results listed in Table 4 and the time series plots of the estimated soil moistures shown in Fig. 5 demonstrate that the soil moisture diagnostic equation with the snow/snowmelt processes included has the capability to capture soil moisture dynamics in snow-dominated regions and can be used to estimate soil moisture in these regions.

Although the average error in the estimated soil moistures is less than 5.0 (%V/V), some RMSEs are greater than 6.0 (%V/V), e.g., the estimated soil moistures in 0–5 cm columns at UT366, UT528 and UT684 in MTP. On the other hand, although the average Nash-Sutcliffe coefficient is 0.72, some Ecs are lower than those of the estimated soil moistures without considering the snow/snowmelt in the SMDE, e.g., at UT684. Are these relatively larger RMSEs and lower Ecs due to the simple method we used in this study to separate rain and snow? As introduced in Section 2, we used 0 °C as the threshold to separate snow and rainfall. Some studies have shown that there exists a probability distribution of snow versus rainfall in the temperature range of 0–7 °C (Dingman, 2002). Auer (1974) utilized about 1000 surface weather observations to produce a simple relationship between the probability of occurrence of the principal precipitation types (rain versus snow) and the surface air temperature (see Fig. 6). Turcotte et al. (2007) proposed another method that utilizes the daily maximum and minimum temperatures to estimate the solid and liquid precipitations as follows:

\[ LP = \begin{cases} 0, & \text{SNOW} = P, \quad \text{if } T_{\text{max}} > 0 \degree \text{C} \\ P \left( \frac{T_{\text{max}}}{T_{\text{max}} - T_{\text{min}}} \right), & \text{SNOW} = P - LP, \quad \text{if } T_{\text{max}} > 0 \degree \text{C} \text{ and } T_{\text{min}} \leq 0 \degree \text{C} \\ P, & \text{SNOW} = 0, \quad \text{if } T_{\text{min}} > 0 \degree \text{C} \end{cases} \]

where LP is the liquid precipitation, P is the total precipitation, SNOW is the solid precipitation (or snow), and \( T_{\text{max}} \) and \( T_{\text{min}} \) are the daily maximum and minimum temperatures in °C, respectively. To test if using the Auer (1974) and Turcotte et al. (2007)'s snow/rain separation schemes can reduce errors in the estimated soil moistures, two additional sets of soil moisture estimations were...
However, comparing the correlation coefficients ($r_B$, equal correlation coefficient compared to the Auer (1974) scheme) always yielded a higher or an equal $r_B$, because the Turcotte et al. (2007) scheme actually is an improvement over the Auer (1974) scheme and may not be able to represent the actual conditions at some sites well, because the Auer (1974) scheme actually is an approximation developed specifically for temperate regions. Using the method described in Section 2, we first estimated the separation method actually was better than the Auer (1974) snow/rain separation method, e.g., at UT853, RMSE of the estimated 0–50 cm soil moisture increased from 5.88 (%V/V) to 7.54 (%V/V) in the PEP. Overall, 38.2% and 39.6% of 72 cases using the daily mean temperature of $0^\circ$C as the single threshold to separate solid and liquid precipitation produced the lowest RMSEs and the highest Nash-Sutcliffe coefficient $E_c$ of the estimated soil moisture using the soil moisture diagnostic equation. The Auer (1974) scheme could not reduce the average error of the estimated soil moistures using the soil moisture diagnostic equation included in the SMDE, and the Nash-Sutcliffe efficiency coefficients ($E_c$) of the estimated soil moisture with and without snow/snowmelt processes are not considered in the SMDE.

The RMSEs, correlation coefficients, and the Nash-Sutcliffe coefficient $E_c$ of the estimated soil moisture using the Auer (1974) scheme and the Turcotte et al. (2007) scheme are 25% (for RMSE) and 25.7% (for $E_c$), and 36.8% (for RMSE) and 34.7% (for $E_c$) for the Turcotte et al. (2007) scheme. These results interestingly show that the simple $0^\circ$C rain/snow separation method actually was better than the Auer (1974) method and slightly better than the Turcotte et al. (2007) scheme, in terms of the accuracy and the Nash-Sutcliffe coefficient of the estimated soil moisture using the soil moisture diagnostic equation. The Auer (1974) scheme could not reduce the average error and improve the correlation coefficient between the estimated and observed soil moisture or the Nash-Sutcliffe coefficient. The main reason is that using a relationship between the probability of occurrence of rain/snow and surface air temperature could improve soil moisture estimation at some sites, but such relationship could be site-specific. The Auer (1974) snow/rain separation scheme may not be able to represent the actual conditions at some other sites well, because the Auer (1974) scheme actually is an averaged relationship between the probability of occurrence of rain/snow and surface air temperature. Therefore, we suggest using $0^\circ$C as the threshold to separate snow and rainfall if no in situ observations of precipitation types are available.

![Fig. 6. Auer's (1974) relationship between the probability of occurrence of the principal precipitation types (rain versus snow) and the surface air temperature.](http://dx.doi.org/10.1016/j.jhydrol.2016.09.063)
Table 5
Estimated optimal parameters in the soil moisture loss function and the associated correlation coefficient between the observed soil moisture values and B values with snow/snowmelt processes and the Auer (1974) or Guttice et al. (2007) snow/rain separation schemes included in the SMDE.

<table>
<thead>
<tr>
<th>Site ID</th>
<th>0–5 cm</th>
<th>0–20 cm</th>
<th>0–50 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>c1</td>
<td>0.242</td>
<td>0.241</td>
<td>0.75</td>
</tr>
<tr>
<td>c2</td>
<td>0.241</td>
<td>0.256</td>
<td>0.70</td>
</tr>
<tr>
<td>c3</td>
<td>0.778</td>
<td>0.776</td>
<td>0.75</td>
</tr>
<tr>
<td>c4</td>
<td>0.780</td>
<td>0.776</td>
<td>0.75</td>
</tr>
<tr>
<td>c5</td>
<td>1.529</td>
<td>1.523</td>
<td>268</td>
</tr>
<tr>
<td>c6</td>
<td>0.74</td>
<td>0.75</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1.529</td>
<td>1.523</td>
<td>268</td>
</tr>
</tbody>
</table>

Table 6
Root mean square errors (RMSE), correlation coefficients (r), and the Nash-Sutcliffe model efficiency coefficients (Ec) of the estimated soil moistures with snow/snowmelt processes and the Auer (1974) or Guttice et al. (2007) snow/rain separation schemes included in the SMDE in the parameter-estimation period (PMP).

<table>
<thead>
<tr>
<th>Site ID</th>
<th>PEP: 0–5 cm</th>
<th>MTP: 0–5 cm</th>
<th>PEP: 0–20 cm</th>
<th>MTP: 0–20 cm</th>
<th>PEP: 0–50 cm</th>
<th>MTP: 0–50 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>0.89</td>
<td>0.63</td>
<td>4.87</td>
<td>0.84</td>
<td>0.50</td>
<td>4.16</td>
</tr>
<tr>
<td>Ec</td>
<td>0.89</td>
<td>0.63</td>
<td>4.87</td>
<td>0.84</td>
<td>0.50</td>
<td>4.16</td>
</tr>
</tbody>
</table>

4.3 Discussion
One advantage of the soil moisture diagnostic equation approach is that errors in the estimated soil moisture are not cumulative (see Table 4). But if we compare the error difference between the parameter estimation period (PEP) and the model-testing period (MTP), we can see that the errors in the MTP usually are slightly higher than those in the PEP. This implies that neglecting the inter-annual variability in the soil moisture loss function could cause some errors in the estimated soil moisture. To solve this problem, we could include other variables such as air temperature, solar radiation, and leaf area index (LAI) in the sinusoidal soil moisture loss function (Pan et al., 2015), which deserves a future study. But nevertheless, the soil moisture diagnostic equation has
shown its ability to estimate root-zone soil moisture in snow-dominated regions and the errors in the estimated soil moistures are not cumulative. This ensures that no re-calibration or re-adjustment of the soil moisture state in the process of estimating soil moisture is needed.

Although the proposed method works well, there is still room for improving the accuracy of the estimated soil moisture through including other factors in the soil moisture loss function and other processes in the soil moisture diagnostic equation. The error sources in the estimated soil moisture using the soil moisture diagnostic equation may be due to the following reasons: (1) the sinusoidal wave function is a first order approximation; including other variables in the loss function may reduce errors. But this will introduce more parameters, and there must be some tradeoff between number of parameters and errors; a systematic study on this issue is needed; (2) snowmelt estimation may have some inaccuracy since we use 0 °C as the threshold to separate snow and rainfall. A site-specific relationship between the probability of occurrence of snow or rain and surface temperature could improve the accuracy of soil moisture estimation in snow-dominated regions; (3) the freeze-thaw cycle in soils is not considered in this study, e.g., rain falling on the frozen ground could not yield infiltration; (4) snowmelt is assumed to start at the bottom of the snowpack and the snowmelt water directly infiltrated into soils; snow sublimation is neglected in this study; (5) the soil moisture diagnostic equation in this study uses a daily time step. During the summer season, precipitation may be primarily delivered to the ground through high intensity, short duration storms. ET is also high during daytime. As a result, soil can wet fast and also dry fast. Relying on a daily time step to estimate soil moisture may not be able to capture the short timescale soil moisture processes and thus induce some errors. These aforementioned error sources deserve a series of future systematic studies, which are beyond the scope of this study.

So far, all studies related to the soil moisture diagnostic equation have used at least a one-year data record of soil moisture measurements to derive the soil moisture loss function parameters and other parameters in the soil moisture diagnostic equation. Although this approach is effective and efficient, it limits application of the soil moisture diagnostic equation to the sites or areas with soil moisture measurements. To eliminate such limitations, additional research is necessary to establish a series of relationships between the parameters in the soil moisture loss function and soil moisture diagnostic equation and other measurable variables, such as climate, latitude, elevation, slope, soil and vegetation characteristics (Coopersmith et al., 2014). In this study we have demonstrated that including snowfall and snowmelt processes in the soil moisture diagnostic equation worked well for estimating soil moisture in snow-dominated regions. So far the soil moisture diagnostic equation has been used in the different climate conditions across the contiguous United States, i.e., humid and semi-humid (Pan, 2012), arid and semi-arid (Pan et al., 2015), and snow-dominated (this study). Therefore, in a future study, we can use almost all soil moisture data collected at all SCAN sites (about 200 sites in the United States, Puerto Rico and Virgin Islands) and SNOTEL sites with soil moisture measurements (about 400 sites in the western United States and Alaska) to establish a series of relationships between the parameters in the soil moisture loss function and soil moisture diagnostic equation and other measurable variables, such as climate, latitude, elevation, slope, soil and vegetation characteristics (e.g., Coopersmith et al., 2015b).

5. Conclusions

In this study, the soil moisture diagnostic equation approach was developed and tested for estimating root zone soil moisture in snow-dominated regions. In the soil moisture diagnostic equation, precipitation is replaced by snowmelt and liquid precipitation (i.e., rainfall). The 0 °C threshold is used for separating snow and rain. The snowmelt is estimated based on the observed snow water equivalent. By including snowfall and snowmelt processes in the soil moisture diagnostic equation, this study showed that the soil moisture diagnostic equation could be used to estimate root zone soil moisture in snow-dominated regions with a relatively good accuracy. A five-water-year (10/1/2010–9/30/2015) dataset of daily precipitation, air temperature, snow water equivalent and soil moistures at three depths (i.e., 5 cm, 20 cm, and 50 cm) at each of 12 Snow Telemetry (SNOTEL) sites across Utah (37.583°N–41.8 83°N, 110.183°W–112.9°W), is applied to test the proposed method. The first three water years are designated as the parameter-estimation period (PEP) and the last two water years are chosen as the model-testing period (MTP). Applying the soil moisture loss function parameters and three empirical parameters in the soil moisture diagnostic equation determined in the PEP, soil moistures in three soil columns (0–5 cm, 0–20 cm, and 0–50 cm) are estimated in the MTP using the soil moisture diagnostic equation. The relatively accurate soil moisture estimations compared to the observations at these 12 SNOTEL sites (RMSE ≤ 0.23%V, average RMSE = 4.28%V/ correlation coefficient ≥ 0.75, average correlation coefficient = 0.89, the Nash-Sutcliffe efficient coefficient Eₜ ≥ 0.24, average Ec ≥ 0.72) indicate that the soil moisture diagnostic equation is capable of accurately estimating soil moisture in snow-dominated regions after the snowfall and snowmelt processes are included in the soil moisture diagnostic equation. The significant advantages associated with the soil moisture diagnostic equation approach are: (1) no initial soil moisture is needed; (2) errors in the estimated soil moistures are not cumulative; (3) thus no recalibration is needed; (4) soil moisture can be estimated in a wide range of thicknesses of the soil column; and (5) the soil moisture diagnostic equation can be used to estimate soil moisture in a wide range of climate zones, e.g., humid and semi-humid, arid and semi-arid, and snow-dominated. Since the approach presented in this study is empirical, the requirement of at least one-year data record of soil moisture for calibration the soil moisture diagnostic equation is the major limitation associated with the soil moisture diagnostic equation approach. To eliminate such limitation, future research is necessary to establish a series of relationships between the parameters in the soil moisture loss function and soil moisture diagnostic equation and other measurable variables, such as climate, latitude, elevation, slope, soil and vegetation characteristics.

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